## 1 Three balls

States that the CM moves with a constant velocity $v/3$	$0.5 \ \mathrm{pts}$
Correctly writes the law of conservation of momentum	
Calculates $v_A$ , $v_B$ and $v_C$ in the CM reference frame	0.9  pts
Correctly writes down kinematic relations for the two rods	
Calculates the kinetic energy $E = mv(2/3)$	0.6  pts
Correctly writes the law of conservation of energy	
Calculates the angular momentum $L = mv\ell$	1.5  pts
Correctly writes the law of conservation of angular momentum.	
Proves the impossibility of the case $\varphi = 0$	1 pt
States that the minimum distance is met when $d\varphi/dt = 0$	$0.5 \ \mathrm{pts}$
Comes to the conclusion that $v_A$ and $v_C$ are perpendicular to AC	
States that at this moment the rods AB and CB have identical angular	1  pt
velocities.	
Uses the relation $I = L^2/(2E)$	1 pt
Uses $L = I\omega$ and $E = I\omega^2/2$	
States that the CM is in the medicenter	0.5  pts
Calculates I as a function of d or $\varphi$	$1 \mathrm{pt}$
Equalizes the two expressions for $I$	$0.5 \ \mathrm{pts}$
Finds the minimum value of $d$	$1 \mathrm{pt}$

## 2 Solenoid

Stating that boiling starts when pressure becomes equal to $p_s$ , the sat-	1 pt
uration pressure (full marks if used correctly implicitly)	
Neglecting $p_s$ as compared to $p_0$ (full marks if used correctly implicitly)	$0.5 \ \mathrm{pts}$
Neglecting water column pressure as compared to $p_0$ (full marks if used	$0.5 \ \mathrm{pts}$
correctly implicitly)	
Concluding that drop due to magnetic forces must be equal to $p_0$ (full	1 pt
marks if used correctly implicitly)	
Showing that $p_0 = p + (\mu_r^{-1} - 1)B^2/(2\mu_0)$	4.5 pt
Partial score for failed attempt: using formula for magnetic field energy	1 pt
density $w = B^2/(2\mu_r\mu_0)$	
interaction energy $\Delta w = (\mu_r^{-1} - 1)B^2/(2\mu_0)$	$1.5 \ \mathrm{pts}$
relating interaction energy difference to pressure difference	2  pts
Alternative approach with dipole-field interaction analysis	
Energy of a magnetic dipole $\vec{d}_m$ in magnetic field $\vec{B}$ : $-\vec{d}_m \cdot \vec{B}$	$0.5 \mathrm{~pts}$
Hence, force acting on a magnetic dipole (parallel to $\hat{x}$ ) in magnetic field	$0.5 \mathrm{~pts}$
(parallel to $\hat{x}$ ): $F = d_m \frac{\mathrm{d}B}{\mathrm{d}x}$	
Induced magnetic dipole moment density: $J = B\chi/(\mu_r \mu_0)$	$0.5 \mathrm{~pts}$
Hence, magnetic force per volume $f_m = B\chi(\mu_r\mu_0)^{-1}\frac{dB}{dx}$	$0.5 \ \mathrm{pts}$
This can be rewritten as $f_m = \frac{1}{2}\chi(\mu_r\mu_0)^{-1}\frac{dB^2}{dx}$	$0.5 \ \mathrm{pts}$
Magnetic force is balanced with the pressure force per volume $f_p = -\frac{dp}{dx}$	$0.5 \ \mathrm{pts}$
Hence $\frac{\mathrm{d}}{\mathrm{d}x} \left[ -p + \frac{1}{2} \chi(\mu_r \mu_0)^{-1} B^2 \right] = \mathrm{const}$	$0.5 \mathrm{~pts}$
Hence $p_0 - p = -\frac{1}{2}\chi(\mu_r\mu_0)^{-1}B^2$	$0.5 \ \mathrm{pts}$
Remark: if the pressure is calculated as a pressure from induced	
solenoidal currents (due to water magnetization) near the side walls of	
test tube, only 2 point out of 4.5 is given (because the pressure at the	
water-wall interface is unknown).	
Using or deriving formula for the magnetic field inside a long solenoid	1 pts
$B = IN\mu_0/\ell$	
Using the above results, expressing $I$	1 pts
Remark: this point can be given only if the solution is correct, except	
for a possible mistake by a factor of $\sqrt{2}$	
Evaluating $I$ numerically	$0.5 \ \mathrm{pts}$
Remark: this point can be given only if the solution is correct, except	
for a possible mistake by a factor of $\sqrt{2}$	

## 3 Staircase

A (2 points in total for part A)

1. 
$$x(n) = n^{2/3}\lambda$$
 (1 pt).

2.  $d_n = x(n+1) - x(n) = \lambda[(n+1)^{2/3} - n^{2/3}]$  (0.5 pt) For  $n \gg 1$ ,  $d_n = (2/3) \lambda n^{-1/3}$  (0.5 pt)

Other solutions leading to the correct answer without computing  $x_n$  are accepted. 0.5 pt for final expression of  $d_n$  are awarded only if both the prefactor and the exponent are correct.

- B (8 points in total for part B)
  - 1. Minimal energy principle expressed mathematically (1 pt)
  - 2. Idea of minimization against small changes in shape (1 pt)
  - 3. Volume conservation principle (2 pt).
  - 4. Energy cost for displacements of one step,  $\epsilon_n(\delta)$  or equivalent, computed correctly (2 pt).
  - 5. Combination of energy minimization and volume conservation in mathematically correct form (1 pt).
  - 6. Correctly derived final answer  $\nu = -2$  (1 pt).